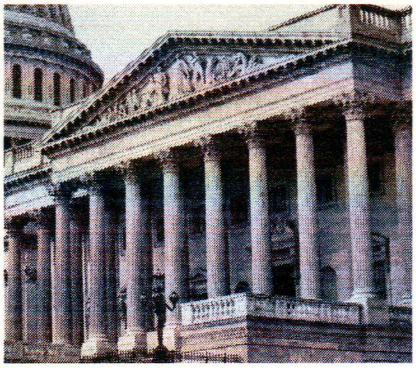
5.5 Special Rights

A Solidify Understanding Task



In previous courses you have studied the Pythagorean theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean theorem? The lengths of two sides

2. What do you need to know about a right triangle in order to use right triangle trigonometry? The length of one side and the measure of one angle (not the 90°)

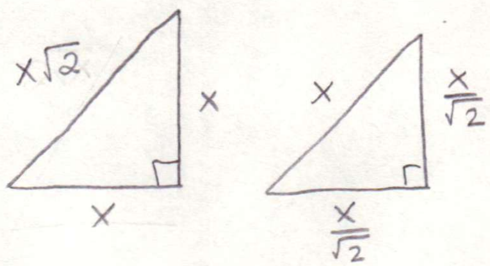
While using the Pythagorean theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trig ratios. There are some right triangles, however, for which knowing a side length and an angle is enough to calculate the value of the other sides without using trigonometry. These are known as *special right triangles* because their side lengths can be found by relating them to another geometric figure for which we know something about its sides.

One type of special right triangle is a 45°-45°-90° triangle.

3. Draw a 45°-45°-90° triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

2 legs are equal   * use a square

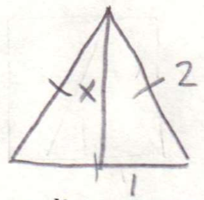
4. Generalize your strategy by letting one side of the triangle measure *x*. Show how the measure of the other two sides can be represented in terms of *x*. (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)



These values can be derived using the pythagorean theorem.

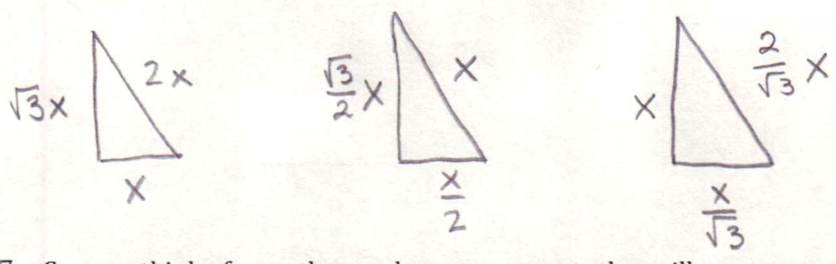
<u>Another type of special right triangle is a 30°-60°-90° triangle</u>.

5. Draw a 30°-60°-90° triangle and assign a specific value to one of its sides. Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.



* use an equilateral triangle

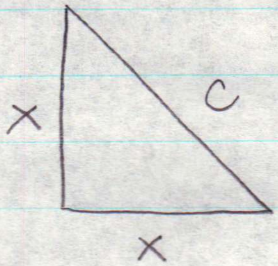
6. Generalize your strategy by letting one side of the triangle measure *x*. Show how the measure of the other two sides can be represented in terms of *x*. (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)



7. Can you think of any other angle measurements that will create a special right triangle?

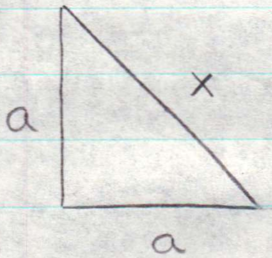
No other special triangles can be found using regular polygons and pythagorean thm.

#4



$$x^2 + x^2 = c^2$$
$$\sqrt{2x^2} = \sqrt{c^2}$$

$$c = \sqrt{2x^2}$$
$$\boxed{c = x\sqrt{2}}$$

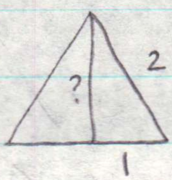


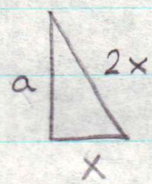
$$a^2 + a^2 = x^2$$
$$\frac{2a^2}{2} = \frac{x^2}{2}$$
$$\sqrt{a^2} = \sqrt{\frac{x^2}{2}}$$

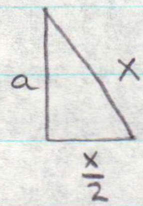
$$a = \frac{\sqrt{x^2}}{\sqrt{2}}$$

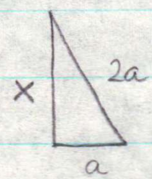
$$\boxed{a = \frac{x}{\sqrt{2}}}$$

5.









$$x^2 + a^2 = (2x)^2$$
$$x^2 + a^2 = 4x^2$$
$$-x^2 \qquad -x^2$$
$$\sqrt{a^2} = \sqrt{3x^2}$$

$$a = x\sqrt{3}$$

$$a^2 + \left(\frac{x}{2}\right)^2 = x^2$$
$$a^2 + \frac{1}{4}x^2 = x^2$$
$$-\frac{1}{4}x^2 \quad -\frac{1}{4}x^2$$
$$a^2 = \frac{3}{4}x^2$$

$$a = \frac{\sqrt{3}}{2}x$$

$$a^2 + x^2 = (2a)^2$$
$$a^2 + x^2 = 4a^2$$
$$-a^2 \qquad -a^2$$
$$\frac{x^2}{3} = \frac{3a^2}{3}$$
$$\sqrt{\frac{1}{3}x^2} = \sqrt{a^2}$$
$$\frac{x}{\sqrt{3}} = a$$